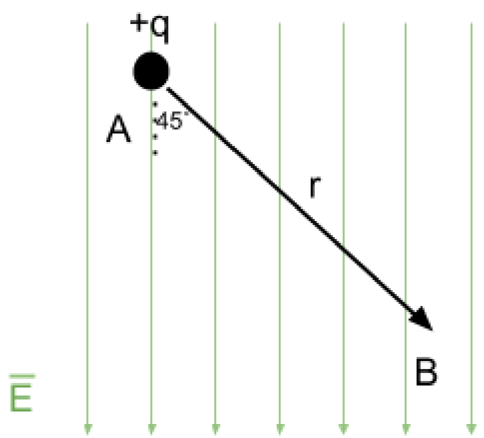
Potential Hard Example (change in potential energy of a charge)

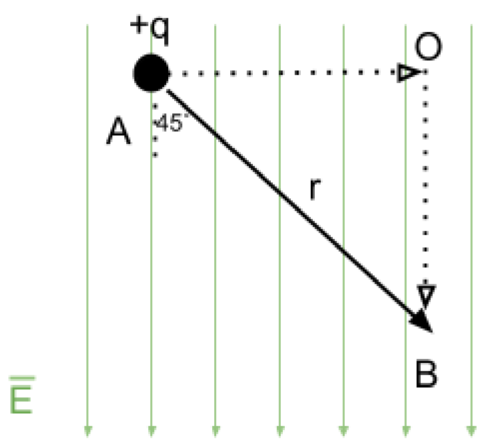
A charge +q is moving along a straight line from A → B, at a 45 degree angle from the direction of a uniform electric field, as illustrated in the diagram below. Calculate the change in electric potential energy of the particle due to the change in position.



Solution

Step 1: Break down the path AB into segments.

One segment AO is perpendicular to the electric field, and the other OB parallel to it:



Step 2: Calculate the potential difference for each path segment.

$$\Delta V = \Delta V_{A\to O} + \Delta V_{O\to B}$$

Since points A and O are on a path perpendicular to the electric field, they are on an *equipotential line*. This means that the potential at both points are the same (i.e. $V_A = V_O$) resulting in
$\Delta V_{A\to O} = V_O - V_A = 0V$

Since points B and O are on a path parallel to the electric field, they have different potential values (i.e. $V_B \neq V_O$). Thus, $\Delta V_{O\to B} = V_B - V_O = -E * r_{OB}$

The negative sign is due to the fact that the positive particle is moving in the direction of electric field; i.e. in the direction of *decreasing potential*.

Step 3: Add up the two segments to get the total potential difference:

$$\Delta V = \Delta V_{A\to O} + \Delta V_{O\to B} = 0 - E * r_{OB} = -E * r_{OB}$$

Plug in any given values.

We are only given the distance from A to B, but in our final equation we need the distance between O and B. To obtain this, we use the geometrical relationship: $r_{OB} = r * cos(45)$

$$\Delta V = -E * r * cos(45) = -\frac{E\,r}{\sqrt{2}}$$

Step 4: Calculate the potential energy using the equation: $\Delta U = q\Delta V$

Answer: the particle losses potential energy as a result of moving from point A to B: $\Delta U = -\frac{qEr}{\sqrt{2}}$